Evaluate $\int_0^\infty y^2 e^{-\frac{y}{2}}\,dy$.

SCORE: _____ / 7 PTS

$= \boxed{\lim_{N\to\infty} \int_0^N y^2 e^{-\frac{y}{2}}\,dy}$ 1

$= \boxed{\lim_{N\to\infty} \boxed{-(2y^2+8y+16)e^{-\frac{y}{2}}}\Big|_0^N}$ 1 2

$= \boxed{\lim_{N\to\infty} \left(-(2N^2+8N+16)e^{-\frac{N}{2}}+16\right)}$ 1

$= -0+16$

$= \boxed{16}$ $\frac{1}{2}$

$\lim_{N\to\infty} \frac{2N^2+8N+16}{e^{\frac{N}{2}}}$ $\frac{\infty}{\infty}$

$= \boxed{\lim_{N\to\infty} \frac{4N+8}{\frac{1}{2}e^{\frac{N}{2}}}}$ $\frac{1}{2}$ $\frac{\infty}{\infty}$

$= \boxed{\lim_{N\to\infty} \frac{4}{\frac{1}{4}e^{\frac{N}{2}}}}$ $\frac{1}{2}$ $\frac{4}{\infty}$

$= \boxed{0}$ $\frac{1}{2}$

u	dv
y^2	$e^{-\frac{y}{2}}$
$2y$	$-2e^{-\frac{y}{2}}$
2	$4e^{-\frac{y}{2}}$
0	$-8e^{-\frac{y}{2}}$

$+$ $-$ $+$

$\int y^2 e^{-\frac{y}{2}}\,dy$

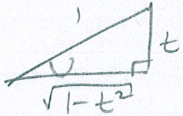
$= -(2y^2+8y+16)e^{-\frac{y}{2}}$

Evaluate $\int e^{\arcsin t}\,dt$. **SCORE: _____ / 8 PTS**

EITHER VERSION OK

1 $\boxed{\begin{aligned} u &= \arcsin t \\ t &= \sin u \end{aligned}}$

$dt = \cos u \, du$



	u		dv
	$\cos u$	$+$	e^u
	$-\sin u$	$-$	e^u
	$-\cos u$	$+$	e^u

1 $\boxed{\int e^u \cos u \, du} = \boxed{e^u \cos u + e^u \sin u - \int e^u \cos u \, du}$ 3

$2\int e^u \cos u \, du = e^u \cos u + e^u \sin u$

$\int e^u \cos u \, du = \boxed{\frac{1}{2} e^u \cos u + \frac{1}{2} e^u \sin u} + C$ 1

$= \boxed{\frac{1}{2} e^{\arcsin t} \sqrt{1-t^2}} + \boxed{\frac{1}{2} t e^{\arcsin t}} + C$

1 1

Evaluate $\int \frac{5x^2 - 20}{x^3 - 4x^2 + 20x}\,dx$.

$$x^3 - 4x^2 + 20x = x(x^2 - 4x + 20)$$
$$= x((x-2)^2 + 16)$$

SCORE: _____ / 8 PTS

$$= \int \left(\boxed{\frac{A}{x} + \frac{B(2x-4)+C}{(x-2)^2+16}} \right) dx = \int \left(\boxed{-\frac{1}{x}} + \frac{\boxed{3(2x-4)}\boxed{+8}}{(x-2)^2+16} \right) dx$$

1 1 1 1

$$\boxed{5x^2 - 20 = A[(x-2)^2 + 16] + B(2x-4)x + Cx}$$ 1

$x=0$: $-20 = 20A \longrightarrow A = -1$

$x=2$: $0 = 16A + 2C \longrightarrow C = -8A = 8$

COEF OF x^2: $5 = A + 2B \longrightarrow B = \frac{1}{2}(5 - A) = 3$

$$= -\ln|x| + 3\ln|x^2-4x+20| + \frac{8}{4}\tan^{-1}\frac{x-2}{4} + C$$

SANITY CHECK:

$x = 3$

$$\frac{45-20}{27-36+60} \stackrel{?}{=} -\frac{1}{3} + \frac{3(2)+8}{17}$$

$$\frac{25}{51} \stackrel{?}{=} -\frac{1}{3} + \frac{14}{17} = \frac{-17+42}{51}$$ ✓ 1

$\frac{1}{2}$

$$= \boxed{-\ln|x|}$$

$\frac{1}{2}$ $\boxed{+ 3\ln|x^2-4x+20|}$

1 $\boxed{+ 2\tan^{-1}\frac{x-2}{4}} + C$

Evaluate $\int_0^5 \frac{12-3z}{z^2-8z+12}\,dz$.

$z^2-8z+12=(z-2)(z-6)$

SCORE: _____ / 7 PTS

$$= \int_0^2 \frac{12-3z}{z^2-8z+12}\,dz + \int_2^5 \frac{12-3z}{z^2-8z+12}\,dz$$

2

$$= \lim_{N\to 2^-} \int_0^N \frac{12-3z}{z^2-8z+12}\,dz$$

1

$$= \lim_{N\to 2^-} \left(-\frac{3}{2}\ln|z^2-8z+12|\right)\Big|_0^N$$

1

$$= \lim_{N\to 2^-} -\frac{3}{2}\left(\ln|N^2-8N+12| - \ln 12\right)$$

1

$$-\frac{3}{2}(-\infty - \ln 12) \to \infty$$

1

$$\int \frac{12-3z}{z^2-8z+12}\,dz$$

$$= -\frac{3}{2}\int \frac{1}{u}\,du \qquad u=z^2-8z+12$$

$$du=(2z-8)\,dz$$

$$= -\frac{3}{2}\ln|u| \qquad -\frac{3}{2}du=(12-3z)\,dz$$

$$= -\frac{3}{2}\ln|z^2-8z+12|$$

INTEGRAL DIVERGES

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